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Radial Flow Filtration for Super-compactible Cakes

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ABSTRACT

Pressure filtration of super-compactible cakes is characterized by negligible increases in both the liquid flow rate and the average percentage of cake solids when the applied pressure drop is above a critical value, which is defined as the critical pressure drop Δp_{cR} (Tiller, F.M.; Li, W.P. Strange behaviour of super-compactible filter cakes. Chem. Process. **2000**, *63* (9), 49) (Tiller, F.M.; Li, W.P. Determination of the critical pressure drop for filtration of super-compactible cakes. Water Sci. Technol. **2001**, *44* (10), 171). When operation pressure is beyond Δp_{cR}, there will be little effect of pressure on either flow rate or the average % cake solids. Lack of understanding of the strange behavior will lead to problems in design and operation of solid/liquid separation systems. In this paper, theory and numerical calculation of radial flow pressure filtration of super-compactible cakes will be presented. Flow rate and average % cake solids as functions of pressure at different cake thickness are calculated for radial flow filtration of flocculated latex. The critical pressure drop Δp_{cR} is determined as a function of cake thickness.

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Key Words: Radial flow filtration; Cake compactibility; Critical pressure drop; Filtrate flow rate; Cake solidosity.

INTRODUCTION

The compactibility of a filter cake is determined by the fragility of the particulate structure and its response to load, and can be classified as incompressible, moderately compactible, and super-compactible. Many materials encountered in industries such as wastewater biosolids, flocculated chemicals, and water treatment residue, fall into the super-compactible category.

In pressure filtration, flow rate and average solidosity can be generally increased by increasing the operating pressure (Kaolin Flat D in Fig. 1). Unexpectedly, for super-compactible materials, experiments and calculations show the existence of a maximum flow rate (Mierlo Biosolids in Fig. 1),[3] and maximum average solidosity (average volume fraction of cake solids). When pressure is beyond a critical value, further increase of pressure will have little effect on either the flow rate or the average solidosity.[2] The critical pressure drop Δp_{cR} is defined as the pressure drop at which the flow rate reaches 90% of its maximum value. The calculated Δp_{cR} is usually small for super-compactible materials, such as 15 kPa for Mierlo Biosolids, 30–50kPa for activated sludge at Houston WWTPs, and only 0.96 kPa for flocculated

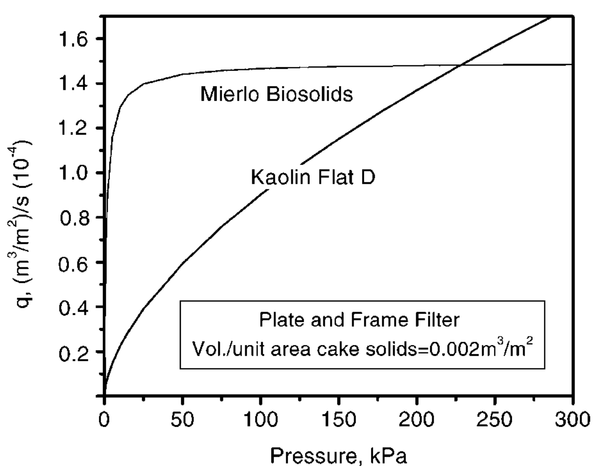

Figure 1. Unexpected behavior of super-compactible cakes.

latex.[1,4] Understanding the unexpected behavior of super-compactible materials in pressure cake filtration is important for SLS system design and operation. In the past few years, theoretical and experimental investigations[1,2,4] on filtration of super-compactible materials have been focused on linear flow filtration as represented by Fig. 1. In this paper, theoretical and numerical results on radial flow filtration for those materials will be presented.

Radial flow cake filtration is encountered in many solid/liquid separation systems such as candle filters (Fig. 2), rotary drum filters, and filtering centrifuges. Flow rate and average cake solidosity are two important parameters for evaluating the performance of filters. Flow rate and average solidosity are calculated as functions of pressure drop at different cake thicknesses for flocculated latex, which gives a dynamic picture of the relationship of flow rate, average cake solidosity, pressure and cake thickness. The critical pressure drop for flocculated latex is determined as a function of cake thickness in radial flow pressure filtration.

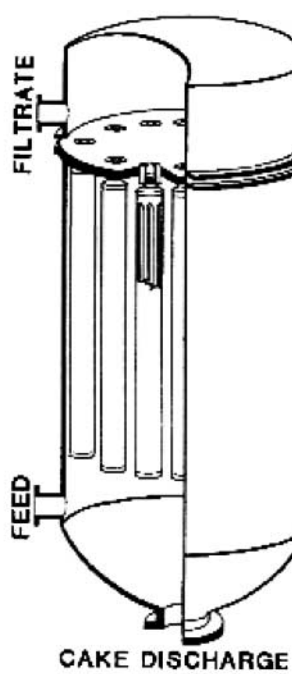

Figure 2. Candle filter with tubular elements.

RADIAL FLOW FILTRATION THEORY

The internal mechanism governing flow and cake compactibility is controlled by the coupled action of the Darcian frictional drag and the compaction of the particulate structure. Development of filtration theory is based on Darcy's law, stress analysis in the particulate matrix, and empirical equations relating local permeability K, and solidosity (volume fraction of cake solids) ε_s to the frictional drag in the cake.

Darcy's Law

For radial flow filtration, the flow rate is related to the pressure gradient and permeability by Darcy's law as

$$\frac{dp_L}{dr} = \frac{\mu q_L}{K} = \frac{\mu Q}{2\pi r K} \tag{1}$$

where p_L = hydraulic or liquid pressure, r = radius in Fig. 3, μ = viscosity, q_L = flow rate per unit area, Q = flow rate per unit height. Although Q is a function of t and r, it is assumed to be independent of r.[5] The local permeability K in Darcy's equation is a function of compressive p_s due to the accumulation of frictional drag force rather than liquid pressure p_L. Stress analysis is necessary to provide the relationship between p_L and p_s.

Stress Analysis

A cake formed externally in radial flow filtration is shown in Fig. 3. Assuming momentum changes due to liquid flow and solid movement are negligible, a free body force balance is applied to relate the hydraulic or liquid pressure p_L to the compressive or effective pressure p_s. The force balance on the element shown in Fig. 3 is

$$(p_L + dp_L + p_s + dp_s)(r + dr)d\theta - (p_L + p_s)(r)d\theta$$
$$- (p_L + k_o p_s)dr \ \ sin(d\theta) = 0 \tag{2}$$

in which k_o is the ratio of the lateral compressive pressure to the radial compressive pressure p_s. The values of k_o generally ranges from 0.4 to 0.7.

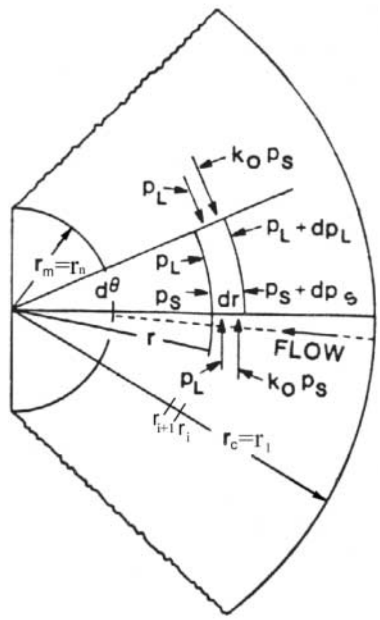

Figure 3. Stresses in a candle filter cake.

Omitting the third order variable in Eq. (2), and rearranging yields

$$d(rp_L)d\theta + d(rp_s)d\theta = (p_L + k_o p_s)dr \ \sin(d\theta) \tag{3}$$

For small angles $d\theta = \sin(d\theta)$, Eq. (3) reduces to

$$\frac{dp_L}{dr} + \frac{dp_s}{dr} + (1 - k_o)\frac{p_s}{r} = 0 \tag{4}$$

The Particulate Structure Equation

Substituting Eq. (4) into Eq. (1) yields the particulate structure equation involving p_s, K, Q and r

$$\frac{dp_s}{dr} + (1 - k_o)\frac{p_s}{r} = -\frac{\mu Q}{2\pi r K} \tag{5}$$

in which K is related to the compressive pressure p_s by empirical equations, and Q is assumed to be independent of r. Combined with empirical equations, Eq. (5) can be solved both analytically and numerically.

EMPIRICAL EQUATIONS RELATING K, ε_S, α TO p_s

The permeability K, solidosity ε_s and specific resistance α can be empirically related to the compressive stress by the following equations,

$$(\varepsilon_s/\varepsilon_{so})^{1/\beta} = (K/K_o)^{-1/\delta} = (\alpha/\alpha_o)^{1/n} = 1 + p_s/p_a \tag{6}$$

where ε_{so}, K_o, α_o, β, δ and p_a are empirical constants. The quantities ε_{so}, α_o, K_o are null-stress values ($p_s = 0$), which correspond to the properties of a freshly deposited layer of cake under no stress. The exponents n, β, δ are known as cake compactibility parameters. They provide a measure of the rate of change of ε_s, α, K with p_s. The relative compactibility of cakes can be classified in accord with the magnitudes of n and δ as follows:

Incompressible	$n = 0$	$\delta = 0$
Moderately compactible	$n \approx 0.4–0.7$	$\delta \approx 0.5–0.9$
Highly compactible	$n \approx 0.7–0.8$	$\delta \approx 0.9–1.0$
Super-compactible	$n > 1$	$\delta > 1.0$

The compactibility parameters for the super-compactible Mierlo Biosolids and moderately compactible Kaolin Flat D in Fig. 2 are shown in Table 1.[6] Another material, flocculated latex, in the table is the most compactible material ever found to the knowledge of authors and will be selected in calculations as follows.

Table 1. Compactibility parameters.

Materials	δ	n	K_o, m^2	α_o, m^{-2}	ε_{so}	p_a, Pa
Kaolin flat D	0.52	0.4	2.4×10^{13}	2.98×10^{13}	0.14	1370
Mierlo biosolids	2.3	1.83	8.3×10^{-12}	4.02×10^{12}	0.03	1000
Flocculated latex	3.484	3.0	1.054×10^{-9}	1.90×10^{10}	0.05	443

NUMERICAL SOLUTION

Calculation Procedure

Combined with empirical equations Eq. (6) and the compactibility parameters, Eq. (5) can be solved numerically. Symbols p, r_c and r_m are used for applied pressure, radius of cake surface, and medium, respectively, in calculation.

Calculation started by dividing the filter cake into n mini units with radius $r_1 = r_c, r_2, r_3, \ldots, r_i, r_{i+1}, \ldots, r_n = r_m$ as shown in Fig. 3. It is assumed that p_s, p_L, K remain constant over small range of radius r_i to r_{i+1}. Integrating Eq. (1) and Eq. (5) from r_i to r_{i+1} yields

$$p_L(i+1) = p_L(i) + \frac{\mu Q}{2\pi K(i)} \ln\left(\frac{r_{i+1}}{r_i}\right) \tag{7}$$

$$p_s(i+1) = p_s(i) + \frac{\mu Q}{2\pi K(i)(1-k_o)} \left(\left(\frac{r_c}{r_{i+1}}\right)^{1-k_o} - \left(\frac{r_c}{r_i}\right)^{1-k_o}\right) \tag{8}$$

In Eqs. (7) and (8), from empirical equations we have

$$K(i) = K_o(1 + p_s(i)/p_a)^{-\delta}, \quad \varepsilon_s(i) = \varepsilon_{so}(1 + p_s(i)/p_a)^{\beta} \tag{9}$$

At the cake surface, i = 1, $p_s = 0$, p_L = applied pressure, $K = K_o$ and $\varepsilon_s = \varepsilon_{so}$. At the medium, i = n, p_s reaches a maximum value, and $p_L = 0$ if the medium resistance is neglected. To start the process, a value of Q is chosen, p_s and p_L are calculated from cake surface to cake medium by Eqs. (7), (8), and (9). In general, p_L at the medium will not be zero under the first assumed Q. The process needs to be repeated until p_L at medium reaches zero when the flow rate Q, inner cake distribution of p_L, p_s, ε_s versus r are solved.

Calculated Results

Numerical calculations illustrating the effect of applied pressure on the radial flow rate Q and the average solidosity ε_{sav} for super-compactible flocculated latex ($\delta = 3.484$, n = 3.0, $K_o = 1.054 \times 10^{-9}$ m^2, $\alpha_o = 1.90 \times 10^{10}$ m^2, and $p_a = 443$ Pa) deposited on a candle filter are shown in Figs. 4 and 5.[7]

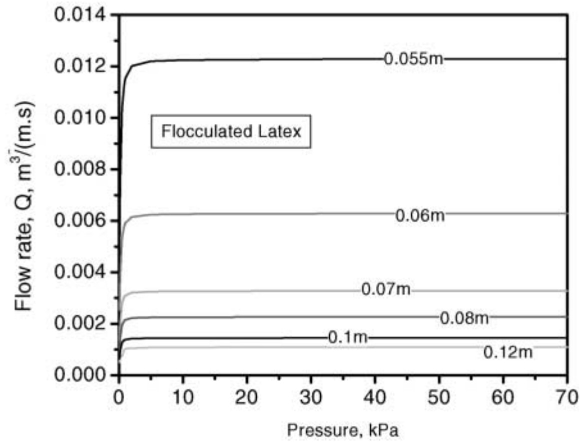

Figure 4. Flow rate vs. pressure.

The radius of candle filter media was $r_m = 0.05$ m, and the cake radius varied as 0.055 m, 0.06 m, 0.07 m, 0.08 m, 0.1 m, and 0.12 m.

In Fig. 4 and Fig. 5, the flow rate and the average solidosity only increase with pressure in the low pressure range. There is negligible effect of pressure

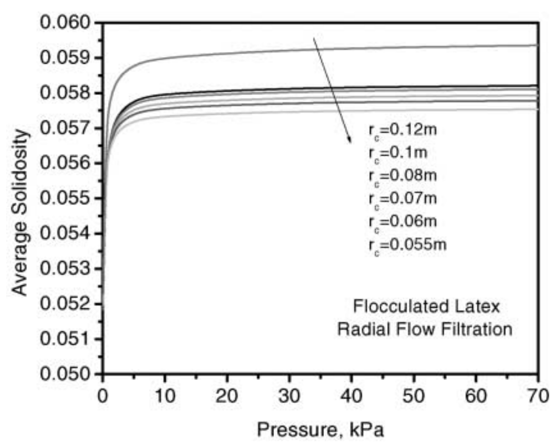

Figure 5. Average solidosity vs. pressure.

Table 2. Critical pressure drop for flocculated latex in radial flow pressure filtration.

r_c, m	0.055	0.06	0.07	0.08	0.1	0.12
Δp_{cR}, Pa	754	735	695	675	618	834

on flow rate and average solidosity when the pressure is increased beyond a critical value. The strange behavior of super-compactible materials in radial flow pressure filtration is the same as in linear flow pressure filtration in Fig. 1. Figures 4 and 5 give a dynamic picture of the relationship of flow rate, average cake solidosity, applied pressure, and cake thickness during radial flow cake filtration operations. If variation of pressure with time is known for each operation, flow rate, cake thickness, average cake solidosity as functions of time can be predicted.

The critical pressure drop Δp_{cR} defined as the pressure at which the flow rate reaches 90% of its maximum value was calculated numerically for cakes of different thickness as shown in Table 2. They are low values smaller than 1000 Pa. The values shown in the table correspond to the critical pressure drop across the cake at each cake thickness.

EXPLANATION OF THE STRANGE BEHAVIOR OF SUPER-COMPACTIBLE CAKES IN RADIAL FLOW FILTRATION

The underlying theory related to the unexpected behavior of super-compactible materials involves the transmission of stress through the particulate bed as reflected by p_L, p_s, and ε_s distribution in a cake of $r_c = 0.055$ m under two different pressures as shown in Figs. 6 and 7. At 500 Pa, which is lower than the critical pressure drop, p_L, p_s, and ε_s vary gradually across the cake. On the other hand, at 50 kPa, which is much higher than the critical pressure drop, p_L, p_s, and ε_s undergo very small change through 90% of the cake and there is a sudden change in a small region near the medium. The concentrated cake layer near the medium is called resistant skin.[8] The skin absorbs most of the pressure drop. After the skin is formed after the pressure is increased beyond the critical pressure drop, further increase of pressure will increase the resistance of the skin, but has little effect on cake solidosity over 90% of cake. Increasing pressure will have little effect on either flow rate or percentage of solids in the cake.

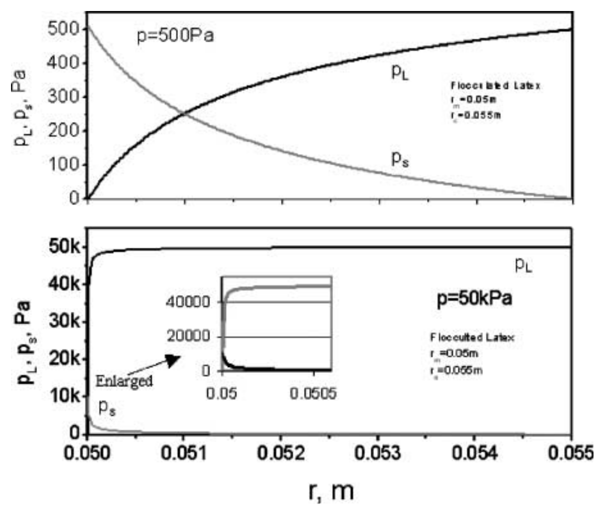

Figure 6. Inner cake p_L p_s distribution.

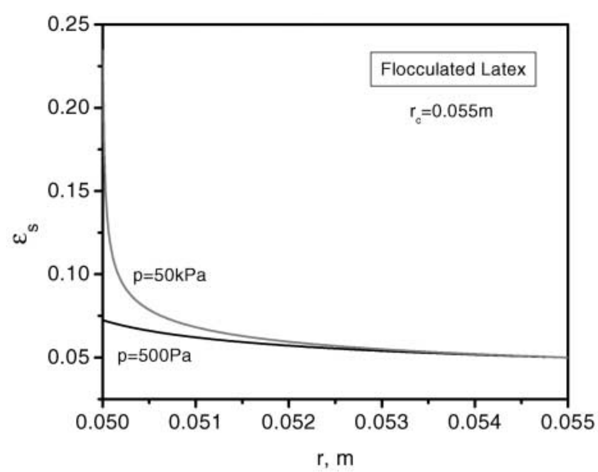

Figure 7. Inner cake solidosity distribution.

CONCLUSION

Compared with moderately compactible materials, the super-compactible materials are characterized by maximum values of flow rate and percentage of cake solids. The critical pressure drop is defined as the pressure at which flow rate reaches 90% of its maximum value. Any operating pressure beyond the critical pressure drop will have little effect on either flow rate or average percentage of cake solids. Based on the theory of radial flow filtration, the flow rate, average solidosity at different pressure and cake thickness for flocculated latex are obtained by numerical calculations. The critical pressure drops at different cake thickness are determined, and are very low values less than 1000 Pa. The underlying explanation of the strange behavior lies in the strange stress and solidosity distribution pattern in the cake and the existence of a concentrated skin near the medium when the applied pressure drop is beyond the critical pressure drop.

NOMENCLATURE

k_o	Ratio of the lateral compressive pressure to the radial compressive pressure in Eq. (2), dimensionless.
K	Cake permeability, m^2
K_o	Null-stress state cake permeability, m^2
n	Cake compressibility parameter in Eq. (6), dimensionless
p_a	Empirical constant in Eq. (6), Pa
p_L	Hydraulic or liquid pressure, Pa
p_s	Solid effective pressure, Pa
Δp_{cR}	The critical pressure drop for super-compactible materials, Pa
q_L	Flow rate per unit area, (m^3/m^2)/sec
Q	Flow rate per unit height, (m^3/m)/sec
r	Radius in Fig. 3, m
r_c	Radius of cake surface in Fig. 3, m
r_m	Radius of filter media in Fig. 3, m
t	Time, sec.
α	Specific cake resistance, m^{-2}
α_o	Null-stress state specific cake resistance, m^{-2}
β	Cake compressibility parameter in Eq. (6), dimensionless
δ	Cake compressibility parameter in Eq. (6), dimensionless
ε_s	Solidosity, which is defined as the fraction of cake solids, dimensionless

ε_{so}	Null-stress solidosity, dimensionless
ε_{sav}	Average solidosity, dimensionless
μ	Liquid viscosity, Pa.s

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